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Application of Quantum Annealer as ILP-solver for the Optimization of Resource Allocation in IP-optical Long-haul Networks

JUPSI (D-Wave Advantage™) Project: QNET

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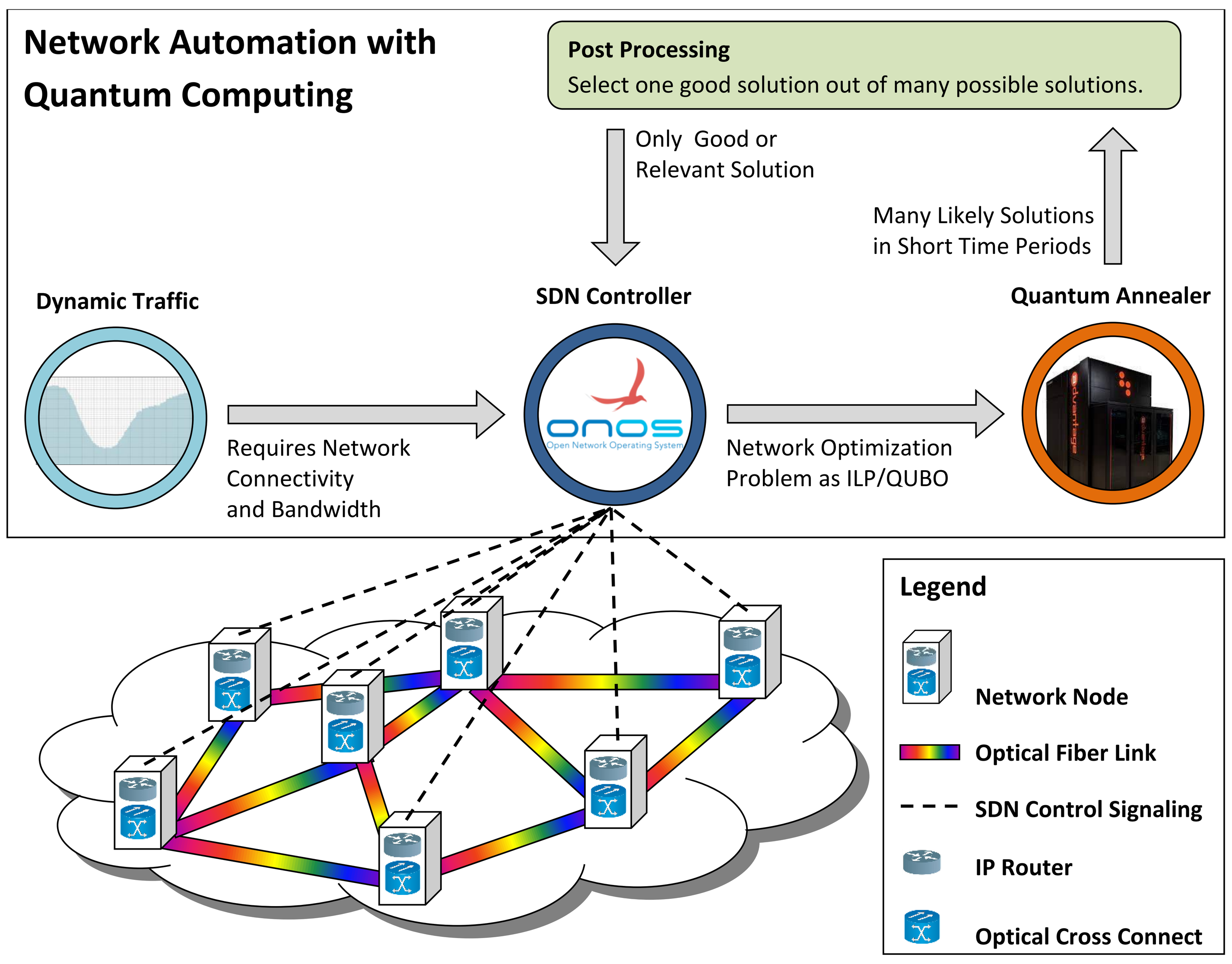
1 Objective



2 Mixed Integer Linear Program for Resource Allocation

Variables:

- Path Selector $\boldsymbol{g} \leftrightarrow g_{d,t_d} \in \{0,1\}$
  $g_{d,t_d} = 1 \leftrightarrow$ Demand d is realized by t_d (else 0)
- Number of active parallel circuits on a path $\boldsymbol{w} \leftrightarrow w_c \in \mathbb{N}$

Objective:

- Minimize total number of all active parallel circuits

$$\min \sum_{c \in C} w_c$$

Constraints:

- Each demand is routed exactly once

$$\sum_{t_d \in T_d} g_{d,t_d} = 1 \quad \forall d \in D$$

- Enough bandwidth is provided to wire all traffic

$$-w_c + \sum_{d \in D} \sum_{t_d \in T_d} H_{c,d,t_d} g_{d,t_d} \leq 0 \quad \forall c \in C$$

$H_{c,d,t_d} \leftrightarrow$ Demand data rate over circuit capacity for circuits which appear in realization

- Enough ports are available to activate required circuits

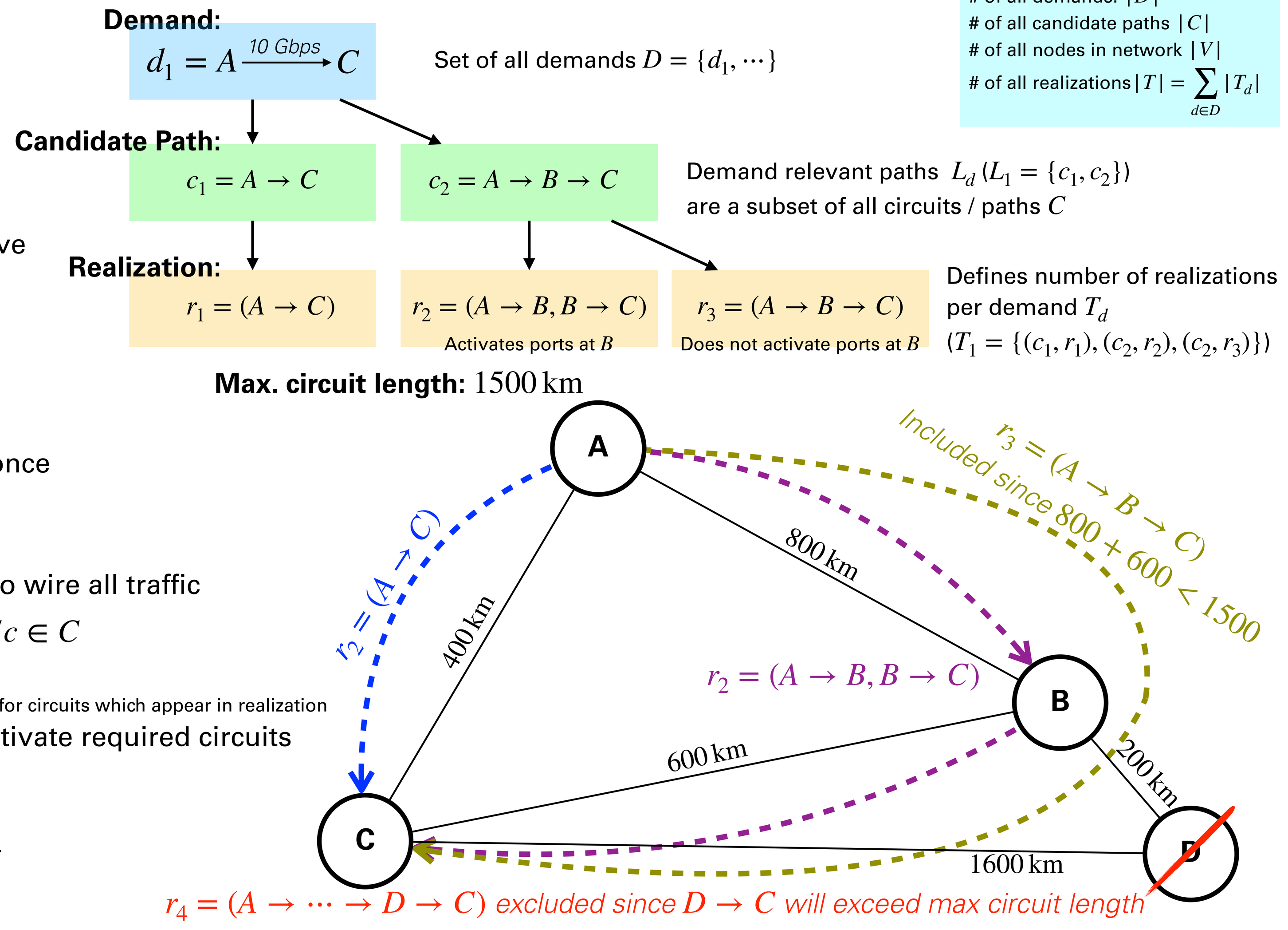
$$\sum_{c \in C} \phi_{v,c} w_c \leq \eta_v \quad \forall v \in V$$

$\phi_{v,c} \leftrightarrow 1$ if v is a source or target note of circuit c

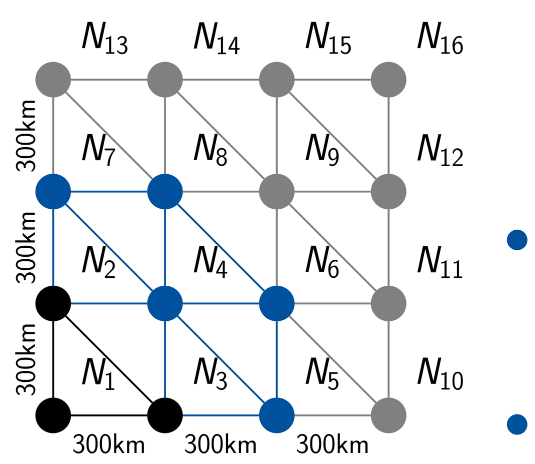
Notation simplified for visualization purposes...

Dimensions:

- # of all demands: $|D|$
- # of all candidate paths $|C|$
- # of all nodes in network $|V|$
- # of all realizations $|T| = \sum_{d \in D} |T_d|$



3 Network Scaling Scenario



$\lvert V \rvert$	3–4	5–7	8–11	12	13–16
η_V	15	31	63	63	127
$\omega_{\max}$	3	3	3	7	7

- Traffic values $H_{c,d,t_d} \sim h_d \left[\frac{\text{Gbit}}{\text{s}}\right]$ are $\mathcal{N}(75, 20)$-distributed
- η_V: installed transceivers
- $\omega_{\max}$: ω's numerical limit

4 Strategy of Problem Mapping

Express m inequalities as equalities via slack

$$A\boldsymbol{x} + \boldsymbol{b} \leq 0\,, \ \boldsymbol{x} \in \mathbb{N}^k\,, \ \boldsymbol{b} \in \mathbb{Z}^m\,, \ A \in \mathbb{Z}^{m \times k}$$

$$\Leftrightarrow \exists \boldsymbol{s} \in \mathbb{Z}^m \geq 0 : A\boldsymbol{x} + \boldsymbol{b} + \boldsymbol{s} = \boldsymbol{0}$$

Quadratic optimization of objective and penalty

$$\boldsymbol{c}^\top \boldsymbol{x} + p \left\| A\boldsymbol{x} + \boldsymbol{b} + \boldsymbol{s} \right\|^2 \to \min$$

Integer encoding for $\boldsymbol{q} \in \{0, 1\}^N$

$$\boldsymbol{x} = Z_x \boldsymbol{q}_x, \quad \boldsymbol{s} = Z_s \boldsymbol{q}_s,$$

Quadratic Unconstraint Binary Opt. (QUBO)

$$X^2(\boldsymbol{q}) = \boldsymbol{c}^\top Z_x \boldsymbol{q}_x + p \left\| A Z_x \boldsymbol{q}_x + \boldsymbol{b} + Z_s \boldsymbol{q}_s \right\|^2 \to \min$$

5 ILP as QUBO Problem

ILP related matrices are of size

$$A = \begin{bmatrix} G^{|D| \times |T|} & 0^{|D| \times |C|} \\ H^{|C| \times |T|} & -\mathbb{I}^{|C| \times |C|} \\ 0^{|V| \times |T|} & \varphi^{|V| \times |C|} \end{bmatrix}, \quad \boldsymbol{b} = \begin{bmatrix} \mathbf{1}^{|D|} \\ \mathbf{0}^{|C|} \\ \boldsymbol{\eta}^{|V|} \end{bmatrix}, \quad \boldsymbol{s} = \begin{bmatrix} \mathbf{0}^{|D|} \\ \boldsymbol{s}_c^{|C|} \\ \boldsymbol{s}_\eta^{|V|} \end{bmatrix} \quad \boldsymbol{x} = \begin{bmatrix} \boldsymbol{g}^{|T|} \\ \boldsymbol{\omega}^{|C|} \end{bmatrix} \quad \boldsymbol{c} = \begin{bmatrix} \mathbf{0}^{|T|} \\ \mathbf{1}^{|C|} \end{bmatrix}$$

Modified objective function in QUBO form becomes

$$X^2(\boldsymbol{q}) = \boldsymbol{q}^\top Q \boldsymbol{q} + C \to \min$$

with $Q = p \begin{bmatrix} Q_{xx} & Q_{xs} \\ Q_{sx} & Q_{ss} \end{bmatrix}, \quad \boldsymbol{q} = \begin{pmatrix} \boldsymbol{q}_x \\ \boldsymbol{q}_s \end{pmatrix}, \quad C = p \|\boldsymbol{b}\|^2$

Subcomponents relate to ILP matrices by

$$Q_{xx} = Z_x^\top A^\top A Z_x + \mathrm{diag}\left\{\left(2\boldsymbol{b}^\top A + \frac{1}{p}\boldsymbol{c}^\top\right) Z_x\right\}$$

$$Q_{ss} = Z_s^\top Z_s + 2\,\mathrm{diag}\left\{Z_s^\top \boldsymbol{b}\right\}, \quad Q_{xs} = Q_{sx}^\top = Z_x^\top A^\top Z_s$$

6 Challenges and Opt. Routes

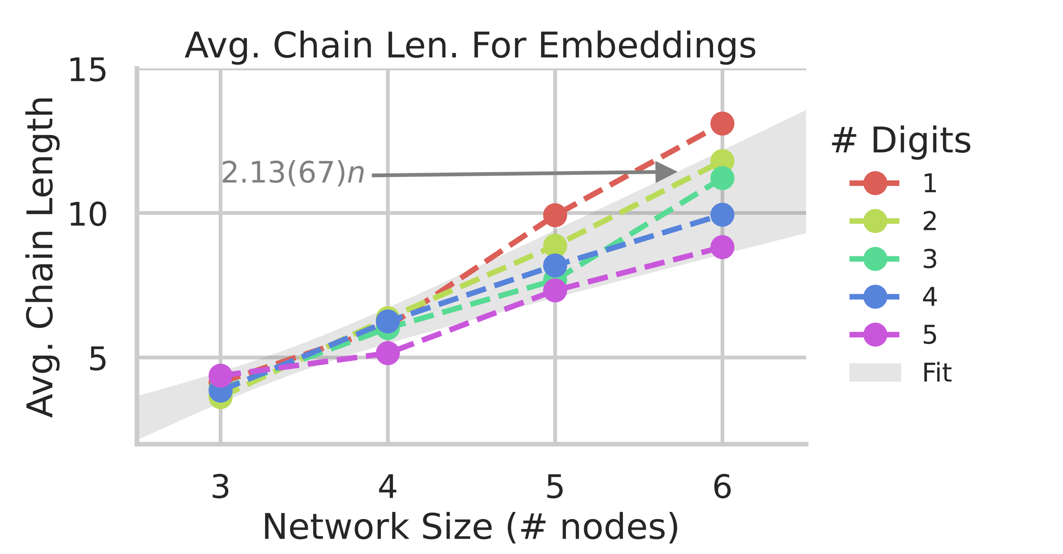
- Finite hardware resources & finding an embedding (once)
  - Available qubits $|q| \lesssim 5.6k$
  - Qubit connectivity $|Q_{i \to j}| \leq 15$ (avg ≈ 14.3) *(For higher connectivity, multiple physical qubits are used to form a logical qubit)*
  - Total connectivity $|\{Q_{ij} \neq 0\}| \lesssim 40.1k$

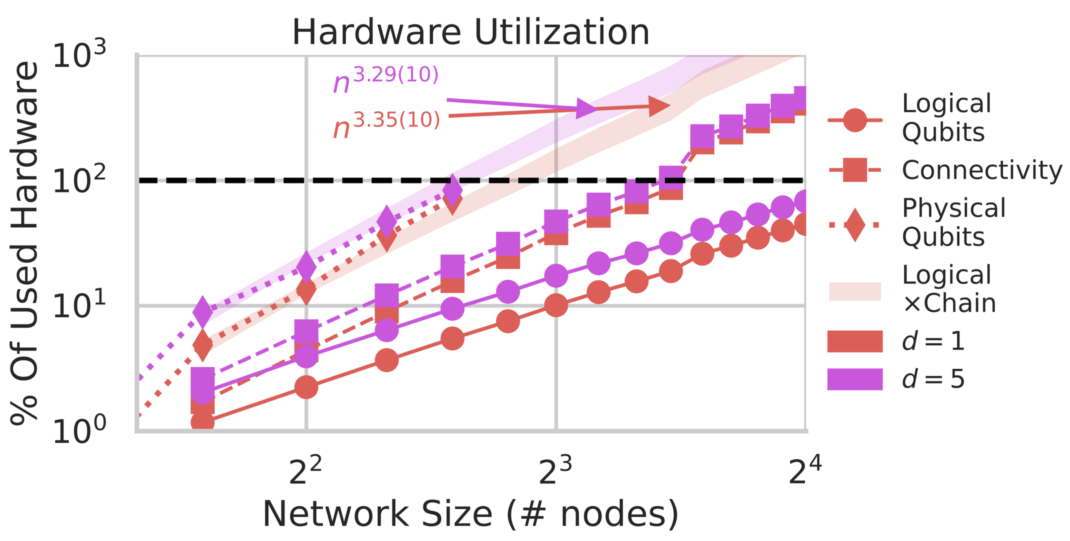
$\Rightarrow$ Minimize slack size by reducing resolution $H \in \mathbb{R}^{|C| \times |T|} \to H \in \mathbb{Q}^{|C| \times |T|}$ *(slack digits)*

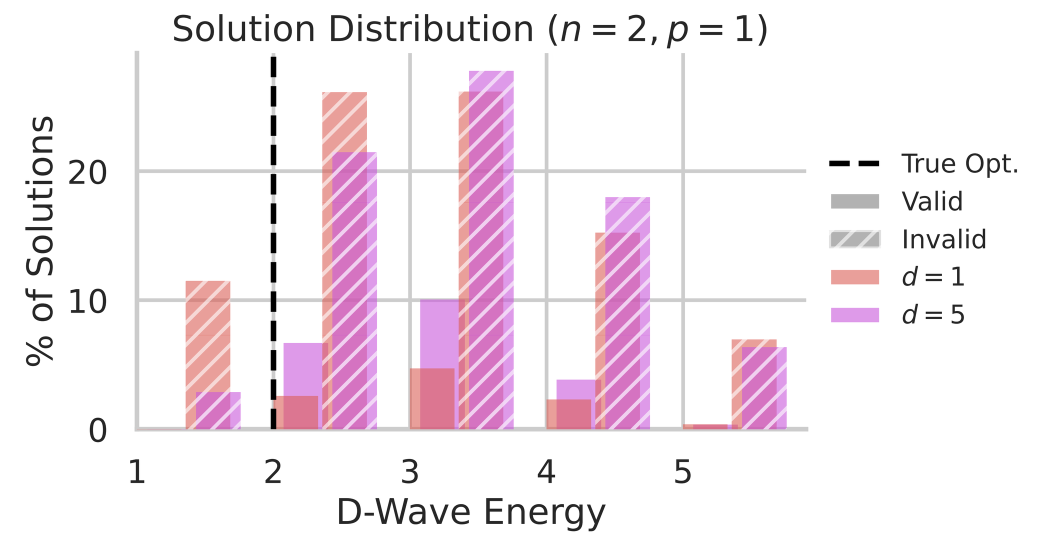
- Finding the ground state *(hardware resolution vs problem energy landscape)*

$\Rightarrow$ Problem resolution dependent penalty term

7 Results







Problem uses same embedding as for $n = 3$. Invalid solutions do not fulfill constraints.
Statistics: $10k$ samples; Annealing time: $1\mu s$.

8 Conclusions

- Theoretical scaling suggests possibility to embed network sizes up to 11 nodes
  Actual embedding related scaling limited to 6 nodes (empirically scaling with $\sim n^{3.3}$):
  Room to improve search for embedding?
- Assuming robust scaling prediction, embedding 15-node networks requires more than $\times 10$ # available qubits
- Possible to find correct solution for smallest possible network with high probability.
  Scaling to larger networks requires further optimizations of algorithm

9 Future Steps

- Embedding search
- Algorithm optimizations
- Hybrid Monte Carlo comparison benchmark
- Open Data access (via EspressoDB)

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