Unterstützt von / Supported by

Alexander von Humboldt
Stiftung/Foundation

Analyzing discretization effects in Nuclear Lattice computations

2020 Fall Meeting of the APS Division of Nuclear Physics | Session LJ.00006

By Christopher Körber
christopher.koerber@rub.de
Ruhr-University Bochum, UC Berkeley & LBNL | Feodor Lynen Fellow

In collaboration with Evan Berkowitz and Thomas Luu

Publication: [1912.04425] (Submitted to PRC)

Source Code & Data: github.com/ckoerber/luescher-nd

Slides: www.ckoerber.com

C. Körber, CC BY-NC 4.0

How large are discretization effects?

Can we control systematic uncertainties if not in the continuum?

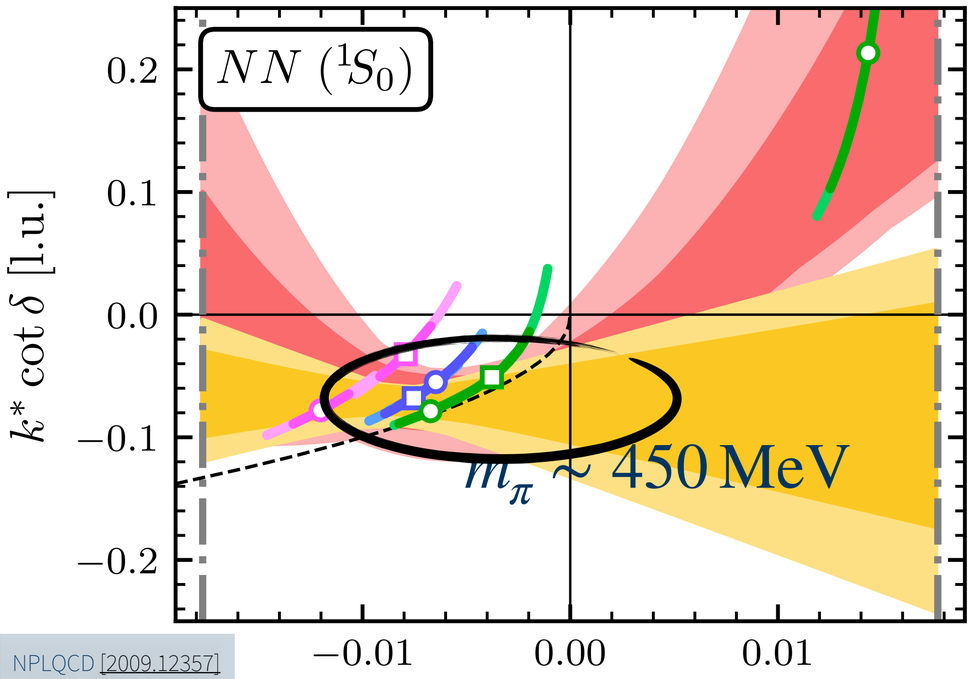
Motivation

Discretization artifacts in (relativistic) Lattice QCD computations?

Example: Different lattice groups find different results in NN sector

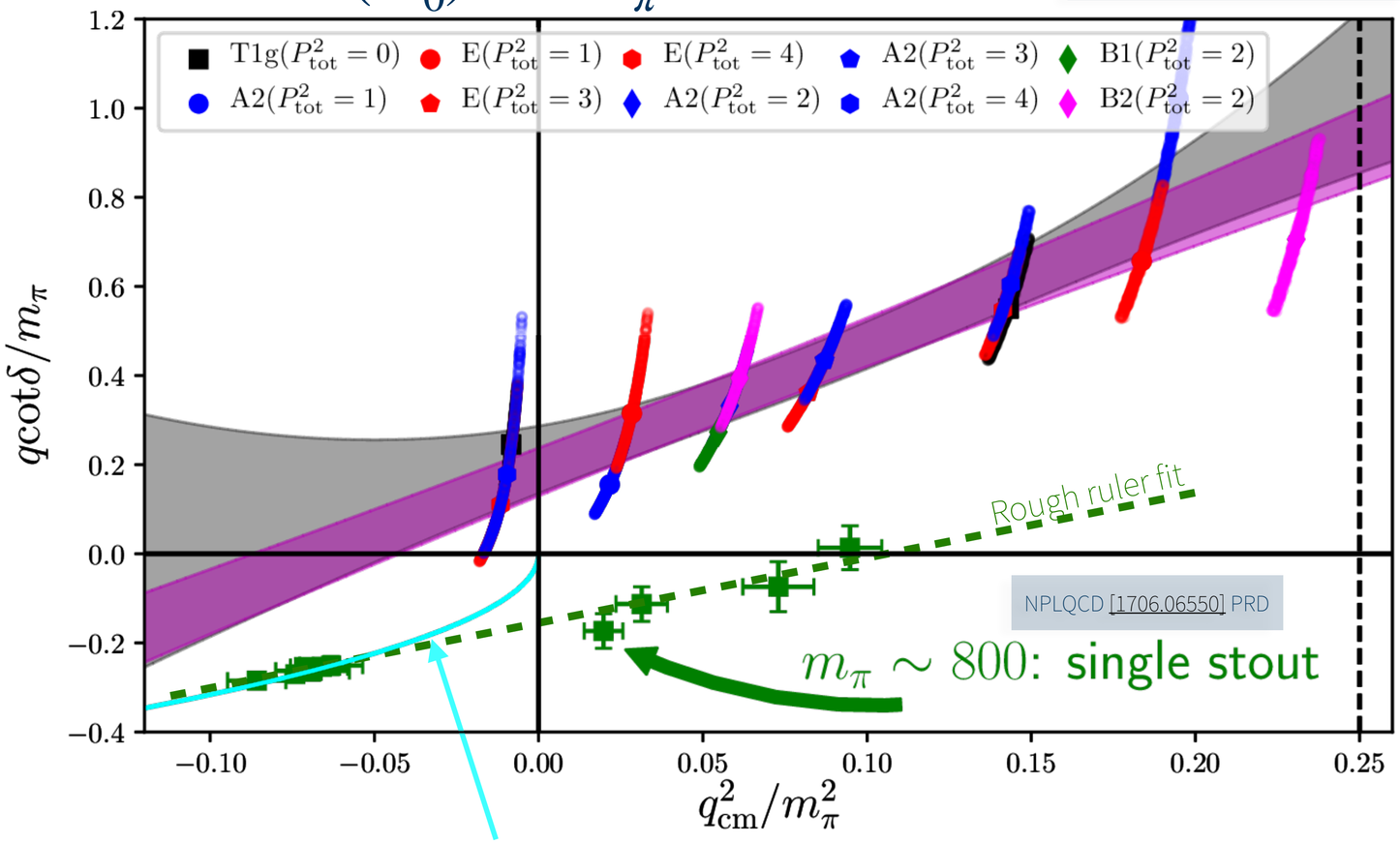
See also:
- HALQCD [1112.5926] Nucl.Phys.A
- T. Yamazaki et al. [1207.4277] PRD
- A. Francis et al. [1805.03966] PRD
- …

- Phase shifts in 1S0 NN channel
- Bound state or not?
- *Observables are in continuum!*
  Does discretization make a difference?



NPLQCD [2009.12357]
Extracted from Fig. 3

Hörz et al. (sLapHnn) [2009.11825]
Extracted from Fig. 4 & 6

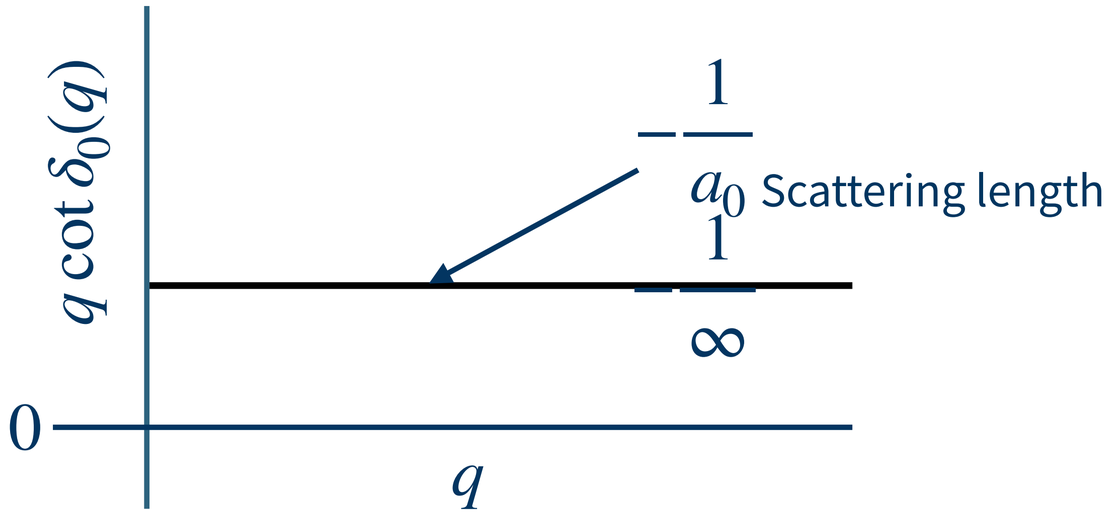


$$k^{*2}\ [\text{l.u.}] \approx (1700\,\text{MeV})^2 \Rightarrow \text{rescale} = \left(\frac{1700}{714}\right)^2 \approx (2.4)^2 \approx 6$$

Motivation

Discretization artifacts in (non-relativistic) Lattice QCD computations?

$$q \cot \delta_0(q) = -\frac{1}{a_0} \to 0$$



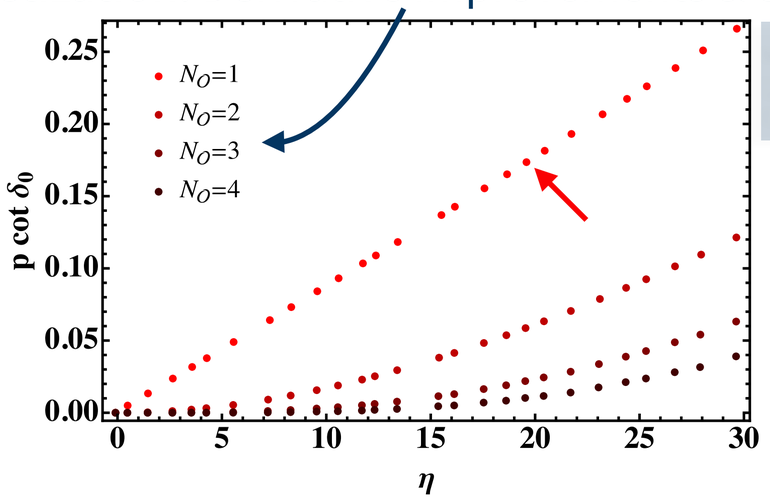
- Describe systems of unitary fermions
  - Scattering independent of scales
  - Scattering length diverges
  - Characterized by short-range interactions
- Similar to nuclear contact interaction in $^1S_0$ np

C. Körber, CC BY-NC 4.0

Motivation

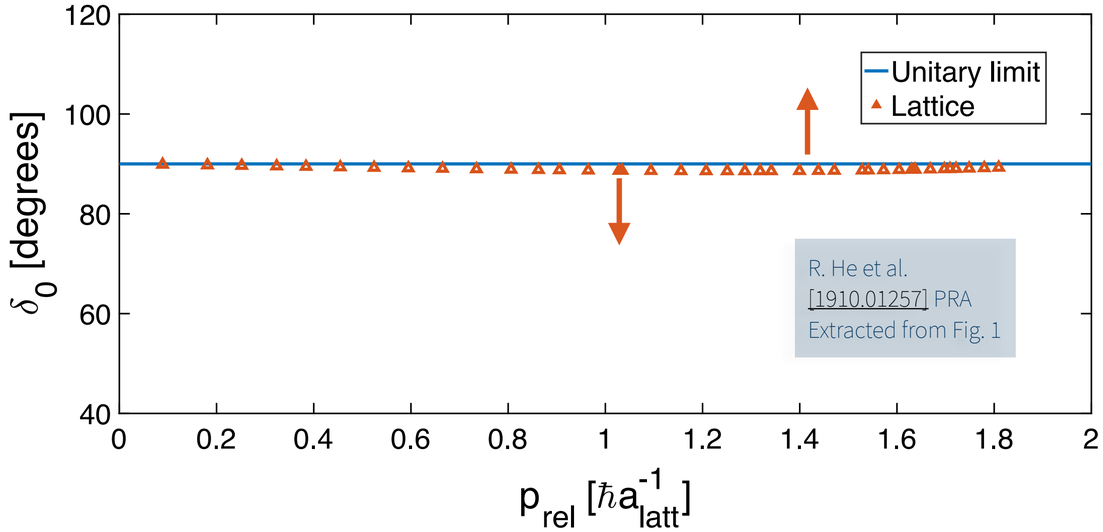
Discretization artifacts in (non-relativistic) Lattice QCD computations?

Discretization: Derivative improvements & continuum limit



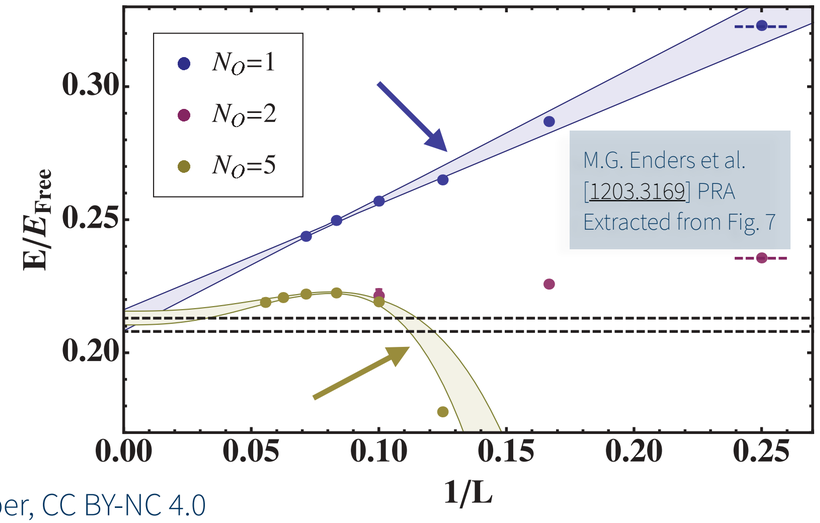
M.G. Enders et al [1106.5725] PRA Extracted from Fig. 2

Discretization: Smeared Hamiltonians & finite spacings



R. He et al. [1910.01257] PRA Extracted from Fig. 1

2+2 Unitary fermions Ground state



M.G. Enders et al. [1203.3169] PRA Extracted from Fig. 7

- At finite lattice spacings, observables depend on discretization details
- Continuum limit can be easier for “less improved discretizations”…
- … even though, at finite spacings, improved schemes can be closer to the continuum

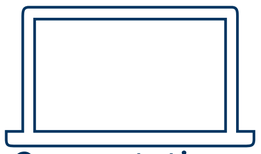
C. Körber, CC BY-NC 4.0

This work

Can we completely control systematic discretization effects for a contact interaction?

Idea

From Finite Volume Lattice to Infinite Volume Continuum



Computations

Finite Volume Discrete Energy Levels Oh(UV & IR)

Fitting coefficients of FV Lattice Hamiltonian to FV C Spectrum

Finite Volume Continuum Energy Levels Oh(IR)

Continuum Limit $a \to 0$

Is there a "discretized Lüscher's formalism"?

$a \to 0 \nearrow L \to \infty$

Standard Lüscher's formalism With finite lattice spacing

Standard Lüscher's formalism

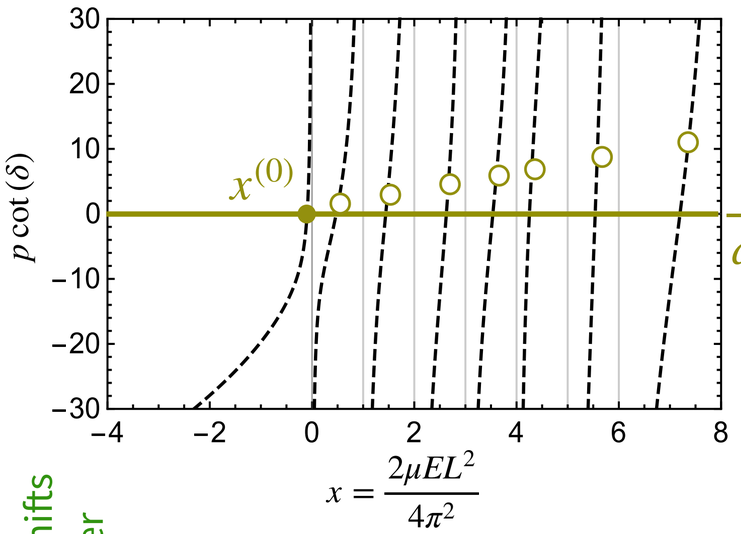
Infinite Volume Discrete Phase Shifts? Oh(UV)

Continuum Limit $a \to 0$

Infinite Volume Continuum Phase Shifts SO(3)

Intersection of Phase Shifts with Standard Lüscher

$\mathcal{Z} : x, L, \Lambda \to p \cot(\delta)$



Lüscher alternatives
- B. Borasoy et al: Spherical Wall [0708.1780] EPJA
- L. Boverman et al: PWD projection[1905.02492] PRC
- K. McElvain at al: HOBET in a Box [1902.03543] PLB
- …

Results of analysis similar to
- R. Seki et al [nucl-th/0509094] PRC

Singular interaction needs regularization

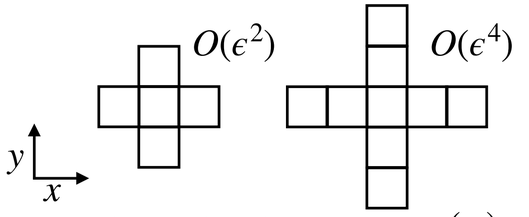
$V(r) = c(\Lambda)\delta(r)$

Does the *"common way"* work?

C. Körber, CC BY-NC 4.0

The spectrum

You need a lot of data…

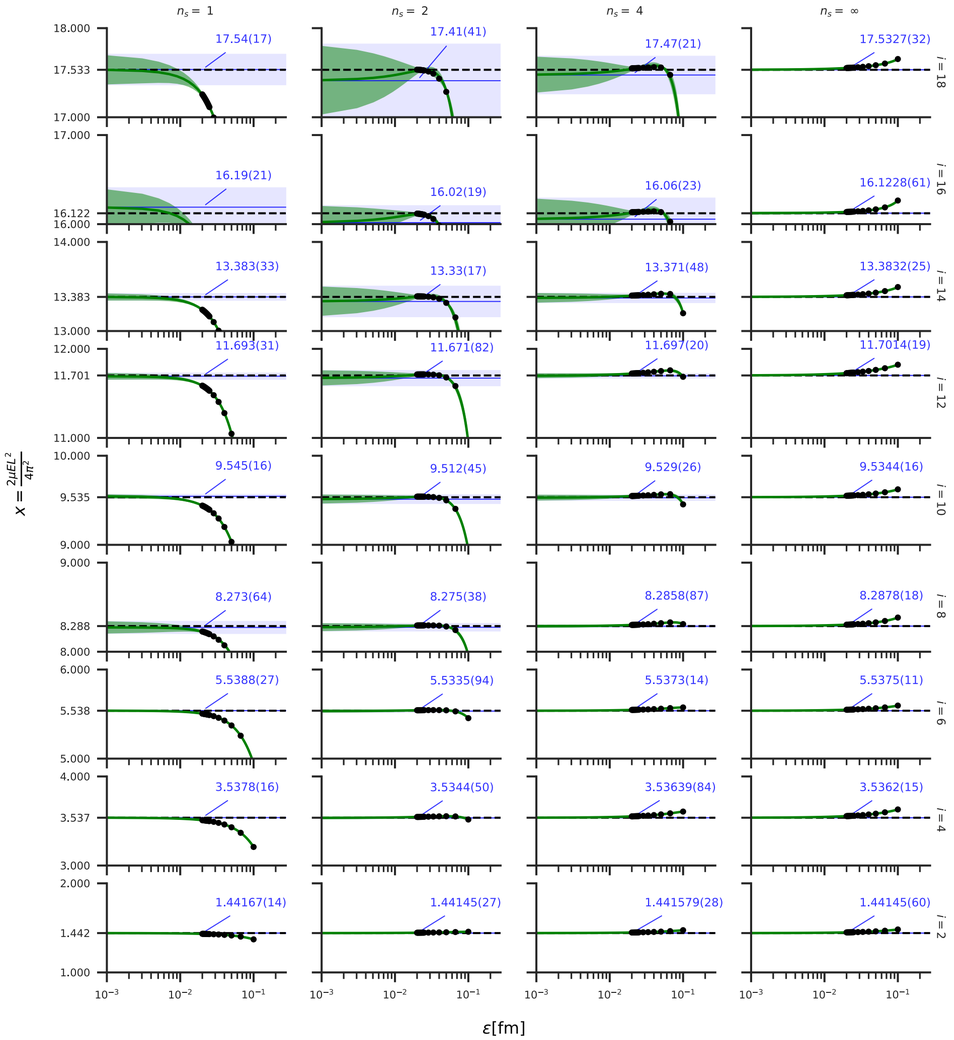


$\cdots$ FT

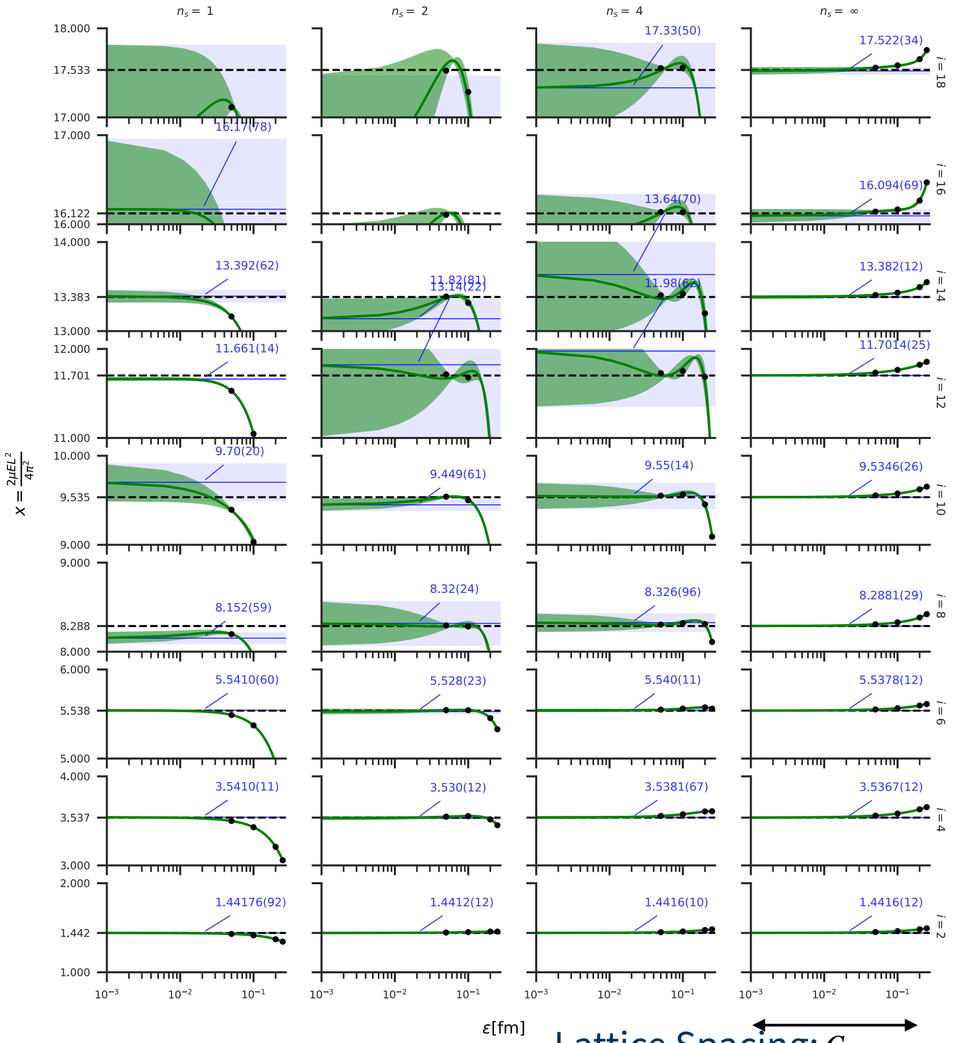
- Level index
- Volume size
- Discretization kind

Discretization type: $\Delta^{(n_s)}$

Volume: $L = 1\text{fm}$



$L = 2\text{fm}$



Level index $x_i(E_i)$

Energy $x(E)$

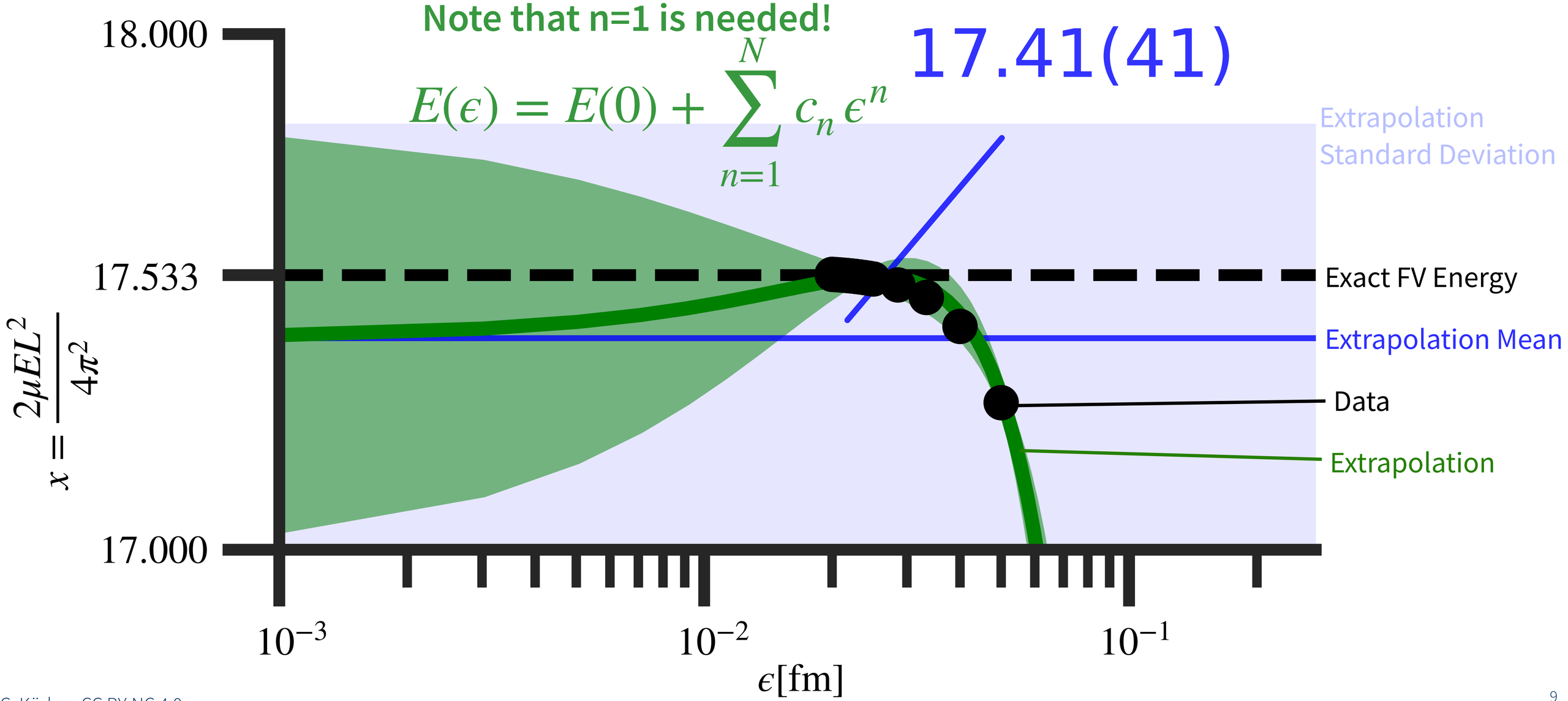
Lattice Spacing: ϵ

C. Körber, CC BY-NC 4.0

The spectrum

Reading frames

$n_s = 2$

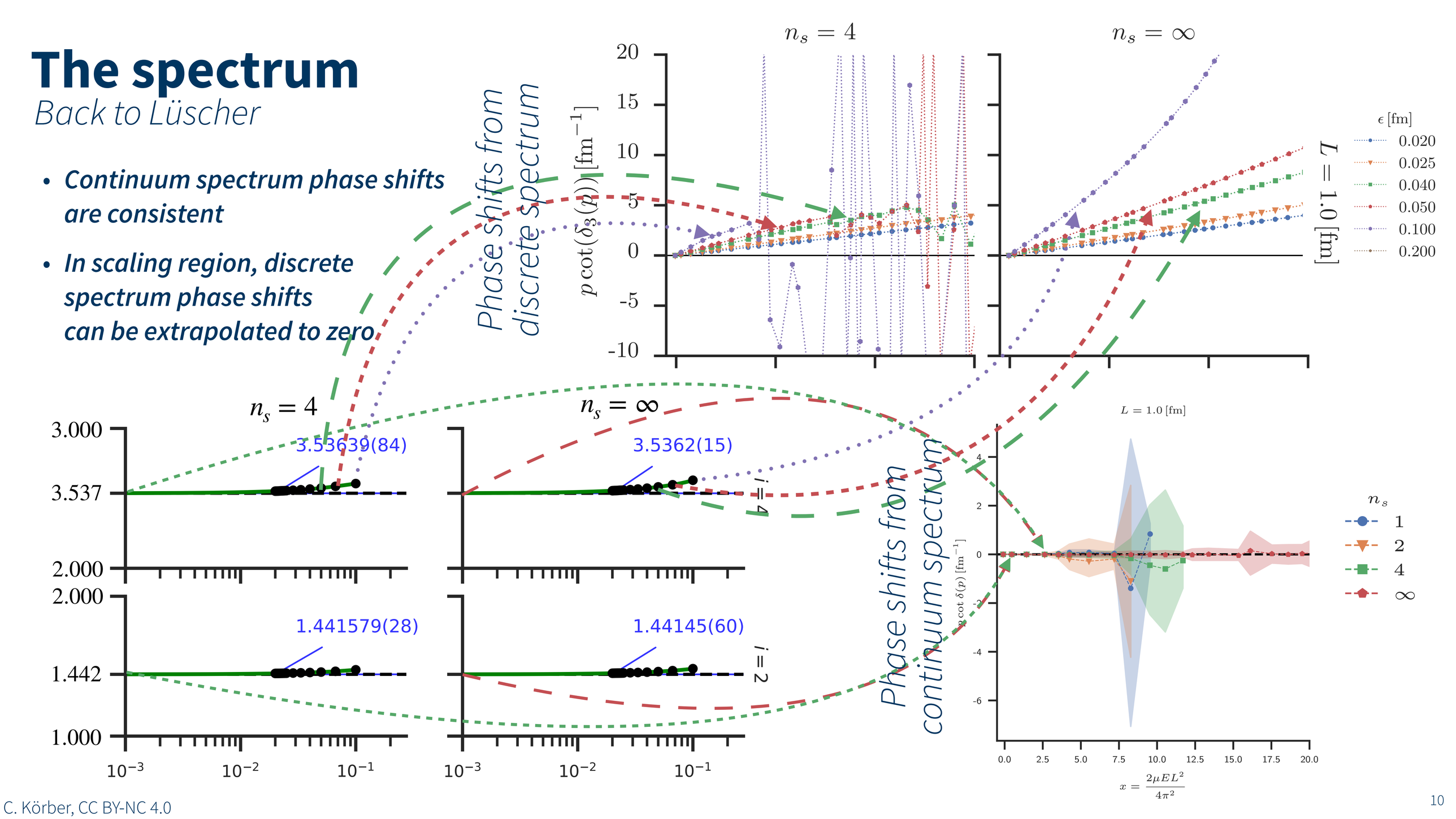


C. Körber, CC BY-NC 4.0

The spectrum

Back to Lüscher

- ***Continuum spectrum phase shifts are consistent***
- ***In scaling region, discrete spectrum phase shifts can be extrapolated to zero***

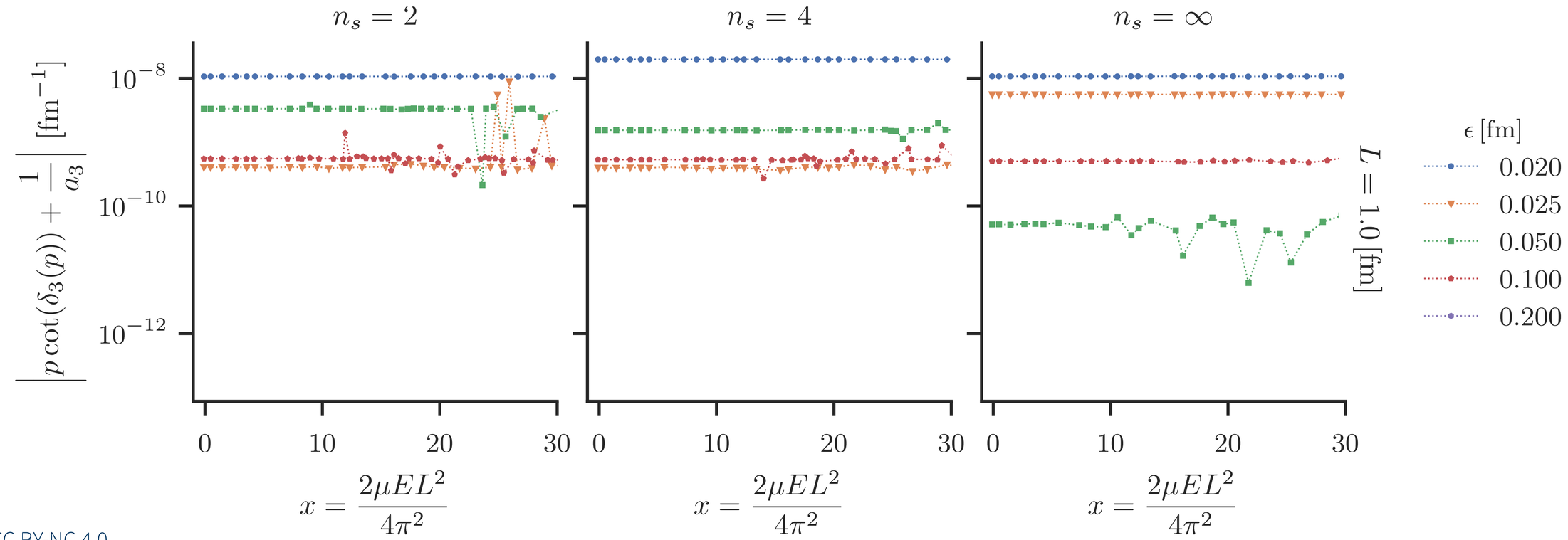


C. Körber, CC BY-NC 4.0

Dispersion zeta

Combined infinite volume and continuum limit

- ***The derivation of Dispersion Lüscher follows the regular derivation except:***
  - The exact lattice momentum dispersion is incorporated
  - The zeta function is evaluated at a finite cutoff matching the lattice spacing
  - We provide more details in the paper [1912.04425]
- ***Phase shifts are entirely for all discretization and any point in the spectrum***



C. Körber, CC BY-NC 4.0

Summary

For a contact interaction…

- ***Sanity check:*** Tuning a contact interaction to FV levels in continuum is consistent **if energy levels are extrapolated to continuum**
- ***Common way:*** First Lüscher then continuum limit works if results in scaling region (discretization under control)
- If in scaling region, both results seem to have comparable precision and accuracy
- Dispersion Lüscher results allow **simultaneous infinite volume and continuum limit**

Open questions:

- *Does this formalism generalize to different interactions:* **No**, but might be perturbatively extendable
- *Do other long-ranged interaction show similar behavior?* Probably yes, if the contact part is dominant
- *How are many-body observables affected?*
- *What does this mean for lattice QCD?*

Thank you for your attention!

By Christopher Körber
christopher.koerber@rub.de

In collaboration with Evan Berkowitz and Thomas Luu

Publication: [1912.04425] (Submitted to PRC) **Source Code & Data:** github.com/ckoerber/luescher-nd **Slides:** www.ckoerber.com

C. Körber, CC BY-NC 4.0