Consistency of Nuclear Forces in the Limit of Large-Nc

01.04.2015 | Christopher Körber | Forschungszentrum Jülich Universität Bonn
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Because of the complexity of phenomena which this theory describes [...] we cannot even dream of solving SU(3) gauge theory exactly. Therefore it is necessary to find some sort of approximation scheme.

— E. Witten, 1979

Quantum Chromodynamics

Quantum Chromodynamics as theory for hadronic interactions is invariant under

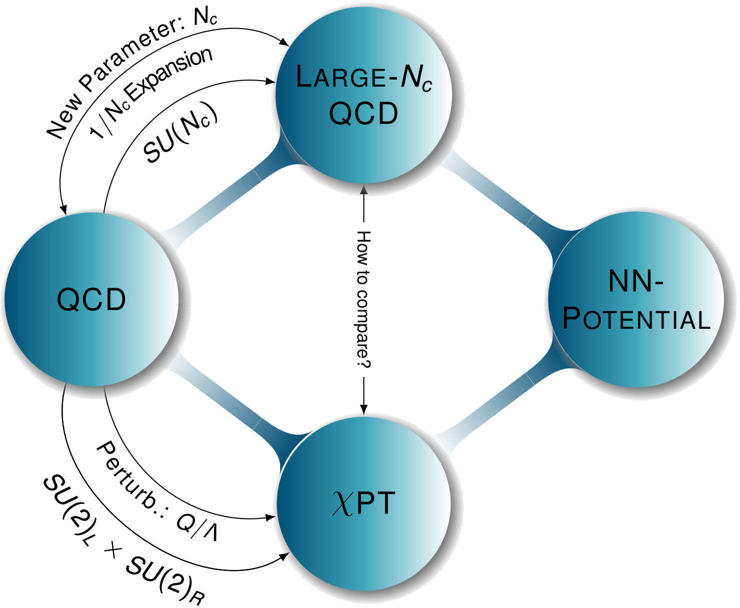
$$SO(3,1)_{Lorentz} \times SU(3)_{flavor} \times SU(3)_{color}$$

- Flavor $SU(3)$ or isospin $SU(2)$ as global symmetry
  - $\rightarrow$ Quarks
- Color $SU(3)$ as local gauge symmetry
  - $\rightarrow$ Gluons
  - $\rightarrow$ perturbative treatment of QCD not possible for energies $< \Lambda_{\mathrm{QCD}}$
- Physical states (hadrons) are colorless compound objects

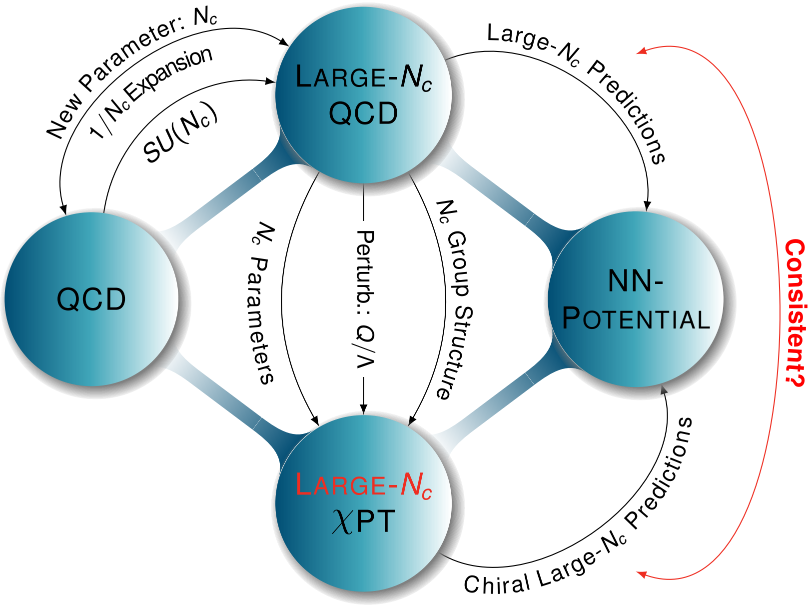
Approximation Schemes for QCD

Possible approximation schemes for QCD

	EFT (e.g. χPT)	Large-N_c QCD
Expansion parameter	Q/Λ_{EFT}	$1/N_c$
Symmetry	$\big(SO(3,1)_L \times SU(3)_f\big) \times \cancel{SU(3)_c}$	$SO(3,1)_L \times SU(3)_f \times SU(N_c)_c$
Degrees of freedom	Hadrons	Quarks and Gluons



New Parameter: N_c
$1/N_c$ Expansion
$SU(N_c)$
LARGE-N_c QCD
QCD
How to compare?
NN-POTENTIAL
χPT
Perturb.: Q/Λ
$SU(2)_L \times SU(2)_R$



LARGE-N_c QCD
QCD
NN-POTENTIAL
LARGE-N_c χPT
New Parameter: N_c
$1/N_c$ Expansion
$SU(N_c)$
Large-N_c Predictions
N_c Parameters
Perturb.: Q/Λ
N_c Group Structure
Chiral Large-N_c Predictions
Consistent?

Motivation

- One already expects the consistency
  - $\rightarrow$ Non-Consistency would indicate that something is wrong
  - $\rightarrow$ Previous analysis has indicated problems at the 3Pion exchange level
- For multiple particles and higher orders in χPT, the number of additional parameters grows
  - $\rightarrow$ Possible to identify importance of contributions within a given order?
- N_c dependence introduces another parameter in χPT
  - $\rightarrow$ Additional information about process in χPT indirectly depending on color?

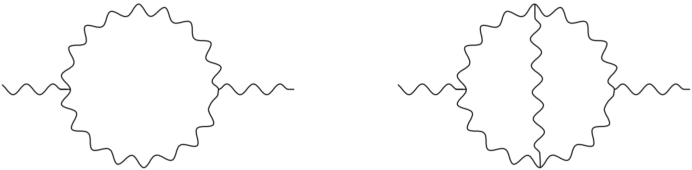
Large-N_c Scaling of Diagrams

Generalize gauge group of QCD: $SU(3)_c \mapsto SU(N_c)_c$

$\Rightarrow$ additional combinatorical factors when computing diagrams

$\rightarrow$ it would be desirable if corresponding diagrams have the same N_c scaling

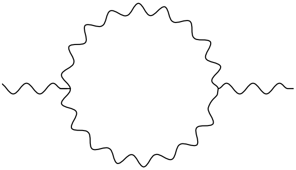
Example: Gluon vacuum polarization diagrams



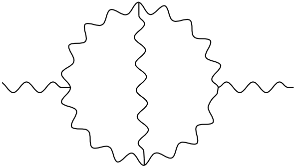
Large-N_c Scaling of Diagrams

t' Hooft 1974

With the usage of structure constants, Casimir invariants, $\cdots$



$$\propto g^2 \sum_{b,c=1}^{N_c^2-1} f^{abc} f^{bcd} = \cdots = g^2 N_c \, \delta^{cd}$$



$$\propto g^4 \sum f^{abc} f^{aeg} f^{beh} f^{ghd} = \cdots = g^4 N_c^2 \, \delta^{cd}$$

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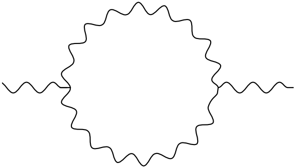

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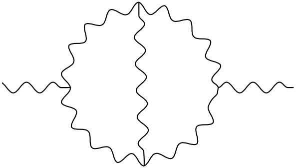


$$\propto g^4 \sum f^{abc} f^{aeg} f^{beh} f^{ghd} = \cdots = g^4 N_c^2 \, \delta^{cd}$$

$\Rightarrow$ the diagrams scale consistent if $g \mapsto g/\sqrt{N_c}$

N_c Scaling of Physical Parameters

Witten 1979
Jenkins 1998

Quantities with N_c scaling

Quantity	Large-N_c scaling	Idea
m_B	N_c	Quark decomposition
$m_N - m_\Delta$	$1/N_c$	Spin-Flavor dependencs
m_π	1	Meson currents
$\vec{q}$	1	Approximation
g_A	N_c	Axial currents
f_π	$\sqrt{N_c}$	Meson currents

$\Rightarrow$ Relativistic expansion q/m_B is $1/N_c$ expansion

$\Rightarrow$ Δ-propagator is equal to nucleon-propagator at leading order

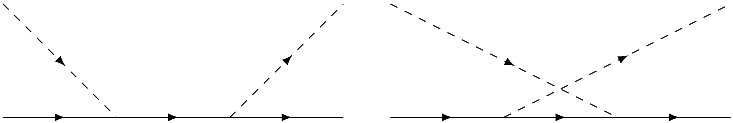
$B + \pi^a \to B + \pi^b$ Scattering

Dashen and Manohar 1993

Pion-Baryon Coupling

$$\frac{\partial_i \pi^a}{f_\pi} \langle B' | \bar{q} \gamma^i \gamma^5 \tau^a q | B \rangle =: \frac{g_A}{f_\pi} \partial_i \pi^a (X^{ia})_{B' B} \propto \frac{N_c}{\sqrt{N_c}}$$

Scaling of $B + \pi^a \to B + \pi^b$ scattering BaryonCouplings



$$N_c \sum_{\tilde{B}} \left[(X^{ia})_{B' \tilde{B}} (X^{jb})_{\tilde{B} B} - (X^{jb})_{B' \tilde{B}} (X^{ia})_{\tilde{B} B} \right] \overset{!}{\sim} N_c^0$$

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Contracted $SU(4)$ Algebra

Dashen, Jenkins, and Manohar 1994

Baryon-meson scattering consistency condition

$$X^{ia} = X_0^{ia} + \mathcal{O}\left(\frac{1}{N_c}\right) \qquad \text{with} \qquad \left[X_0^{ia} \, , \, X_0^{jb} \right] = 0$$

Including spin and isospin operators generates the contracted $SU(4)$ algebra

Contracted $SU(4)$ Algebra

$$\left[J^i \, , \, J^j \right] = i\,\epsilon^{ijk} J^k \, , \qquad \left[I^a \, , \, I^b \right] = i\,\epsilon^{abc} I^c \, , \qquad \left[I^a \, , \, J^i \right] = 0 \, ,$$

$$\left[J^i \, , \, X_0^{jb} \right] = i\,\epsilon^{ijk} X_0^{kb} \, , \qquad \left[I^a \, , \, X_0^{jb} \right] = i\,\epsilon^{abc} X_0^{jc} \, , \qquad \left[X_0^{ia} \, , \, X_0^{jb} \right] = 0$$

Further analysis

$$\left[X^{ia} \, , \, X^{jb} \right] \sim 1/N_c^2$$

Nucleon-Nucleon Potential

Kaplan and Savage 1996
Kaplan and Manohar 1997

General form for central nucleon-nucleon potential

$$V_{NN}^{C} = V_0^0 + V_\sigma^0 \, J_1 \cdot J_2 + V_0^1 \, I_1 \cdot I_2 + V_\sigma^1 \, X_1 \cdot X_2$$

N_c scaling of potential $\leftrightarrow$ one-quark operator decomposition

$$V_{NN}^{C} = N_c \sum_{n, n_1, n_2} V_{n_1 n_2}^{(n)} \left(q^2\right) \left\langle N_1', N_2' \left| \left(\frac{\hat{J}}{N_c}\right)^{n_1} \cdot \left(\frac{\hat{I}}{N_c}\right)^{n_2} \cdot \left(\frac{\hat{X}}{N_c}\right)^{n - n_1 - n_2} \right| N_1, N_2 \right\rangle$$

contractions of operators can be reduced using representation theory

$$\left\langle N' \middle| \hat{I} \middle| N \right\rangle \sim N_c^0 \quad , \quad \left\langle N' \middle| \hat{J} \middle| N \right\rangle \sim N_c^0 \quad , \quad \left\langle N' \middle| \hat{X} \middle| N \right\rangle \sim N_c$$

Confirmation of $I_t = J_t$ Rule

	V_0	V_σ	V_T
$\mathbb{1}_1 \cdot \mathbb{1}_2$	N_c	$1/N_c$	$1/N_c$
$I_1 \cdot I_2$	$1/N_c$	N_c	N_c

Chiral Symmetry in QCD

Starting with QCD for light up and down quark

$$\mathscr{L}_{QCD} = \bar{q}\left(i\not{D} - m\right) q - \frac{1}{4}\mathrm{Tr}\left[\mathcal{G}_{\mu\nu}\mathcal{G}^{\mu\nu}\right]$$

Introduce chiral projection operators

$$P_R := \frac{\mathbb{1} + \gamma^5}{2}, \qquad P_R\, q =: q_R\,, \qquad P_L := \frac{\mathbb{1} - \gamma^5}{2}, \qquad P_L\, q =: q_L$$

Formulate Lagrangian with chiral quark components

$$\mathscr{L}_{QCD} = \bar{q}_L\left(i\not{D}\right) q_L + \bar{q}_R\left(i\not{D}\right) q_R - \bar{q}_R m\, q_L - \bar{q}_L m\, q_R - \frac{1}{4}\mathrm{Tr}\left[\mathcal{G}_{\mu\nu}\mathcal{G}^{\mu\nu}\right]$$

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Formulate Lagrangian with chiral quark components

$$\mathscr{L}_{QCD} = \bar{q}_L\left(i\not{D}\right) q_L + \bar{q}_R\left(i\not{D}\right) q_R - \cancel{\bar{q}_R m\, q_L - \bar{q}_L m\, q_R} - \frac{1}{4}\mathrm{Tr}\left[\mathcal{G}_{\mu\nu}\mathcal{G}^{\mu\nu}\right]$$

Consider chiral approximation: $m = \mathrm{diag}(m_u, m_d) \to 0$

$\Rightarrow$ $\mathscr{L}_{QCD}$ is invariant under $SU(2)_L \times SU(2)_R$

Pion-Nucleon Lagrangian

Weinberg 1979
Gasser and Leutwyler 1984

- Change degrees of freedom:

$$(\mathrm{q}_{\mathrm{up}}, \mathrm{q}_{\mathrm{down}}, \text{gluons}) \longrightarrow (\text{Nucleons, Pions, } \cdots)$$

- Create most general Lagrangian with new degrees of freedom:
  $\rightarrow$ new Lagrangian has to respect the symmetries of $\mathscr{L}_{QCD}$

$$\mathscr{L}_{\chi} = \mathscr{L}_{N\pi}^{(1)} + \mathscr{L}_{\pi}^{(2)} + \cdots$$

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# of derivatives

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contained degrees of freedom

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\# of derivatives

$$\mathscr{L}_\chi = \mathscr{L}_{N\pi}^{(1)} + \mathscr{L}_{\pi}^{(2)} + \cdots$$

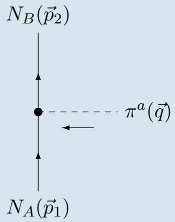
contained degrees of freedom

Extract lowest order pion-nucleon interactions

$$\mathscr{H}_{21}(x) = \frac{g_A}{2f_\pi} N^\dagger(x)\, X^{ia} \left(\partial^i \pi^a(x)\right) N(x)$$

\# of nucleon legs   \# of pion legs

$\mathscr{H}_{21}$-Vertex



Nucleon-Nucleon Potential in ChPT

Okubo 1954
Weinberg 1991
Bernard, Kaiser, and Meißner 1995
Epelbaum 2007

Compute the 2N-potential using:

- Method of unitary transformation

$$V_{\text{eff}}^{\text{UT}} = \eta \left(\mathbb{1} + A^\dagger A \right)^{-1/2} \left(H + A^\dagger H + HA + A^\dagger HA \right) \left(\mathbb{1} + A^\dagger A \right)^{-1/2} \eta - H_0$$

$$\lambda A \eta = \frac{\lambda}{E_\eta - E_\lambda} \left(H - [\, A \,,\, H \,] - AHA \right) \eta \,, \quad E_\eta := \eta H_0 \eta \,, \quad E_\lambda := \lambda H_0 \lambda$$

- ChPT powercounting

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Nucleon-Nucleon Potential in ChPT

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Compute the 2N-potential using:

- Method of unitary transformation $\to$ iterative equation for potential

$$V_{\text{eff}}^{\text{UT}} = \eta \left(\mathbb{1} + A^\dagger A \right)^{-1/2} \left(H + A^\dagger H + HA + A^\dagger HA \right) \left(\mathbb{1} + A^\dagger A \right)^{-1/2} \eta - H_0$$

$$\lambda A \eta = \frac{\lambda}{E_\eta - E_\lambda} \left(H - [\, A \,,\, H \,] - AHA \right) \eta \,, \quad E_\eta := \eta H_0 \eta \,, \quad E_\lambda := \lambda H_0 \lambda$$

- ChPT powercounting $\to$ systematic perturbation for equation

$$\nu = 4 - 3N + \sum V_i \kappa_i \qquad \text{with} \qquad \kappa_i = d_i + p_i + \frac{3}{2} n_i - 4$$

$$V_{\text{eff}}^{\text{UT}\,(\nu)} = f(H^{(\kappa)}, A^{(\kappa)})$$

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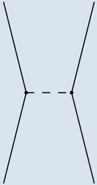
Results

Banerjee, Cohen, and Gelman 2002

- One pion exchange

Leading order: trivially consistent



$$V_{\mathrm{UT}}^{\mathrm{eff}\,(0)} = -\eta\, H_{21}^{(1)} \frac{\lambda}{\omega_\pi} H_{21}^{(1)}\, \eta$$

$$= -\frac{g_A^2}{4 f_\pi^2} \frac{X_1^{ia}\, q^i\, X_2^{ja}\, q^j}{\vec{q}^{\,2} + m_\pi^2} \sim N_c$$

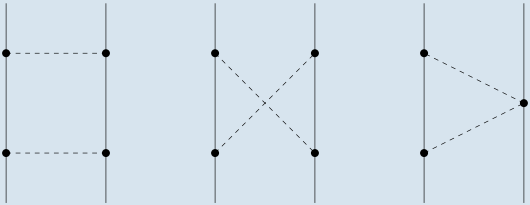
- Two pion exchange level
- Three pion exchange level

Results

Banerjee, Cohen, and Gelman 2002

- One pion exchange
- Two pion exchange level

Two pion exchange insights



- Operator structure essential for consistency $\rightarrow$ internal Δs required
- Order scales as $1/N_c$ ($1/N_c^2$ suppressed compared to ChPT LO)

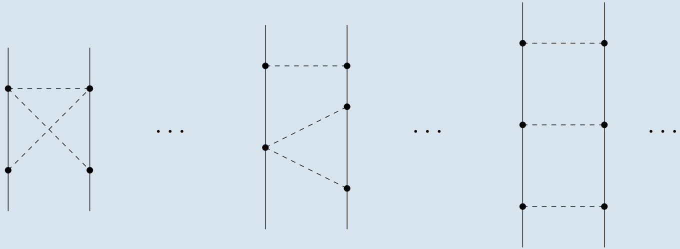
- Three pion exchange level

Results

Banerjee, Cohen, and Gelman 2002

- One pion exchange
- Two pion exchange level
- Three pion exchange level

Additional insights



- Unitary structure essential for satisfying consistency
- Order contributes to N_c as well as $1/N_c$

Future Motivation

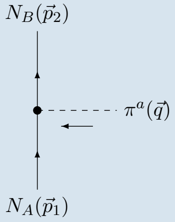
- Include NN-contact interactions
  - $\rightarrow$ Contact interactions also consitent
  - $\rightarrow$ Importance and dependence of interactions (do some interactions collectively contribute to lower orders in N_c?)
- Include further process (e.g. 3N-forces) in analysis
- Generalize to proof for all orders
  - $\rightarrow$ Problematic: higher orders do not only depend on previous orders (induction difficult)

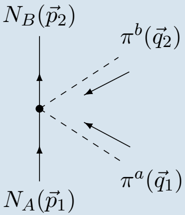
End

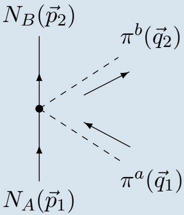
Thank you for your attention

Feynman Rules

Feynman Rules for Lowest Order Vertices







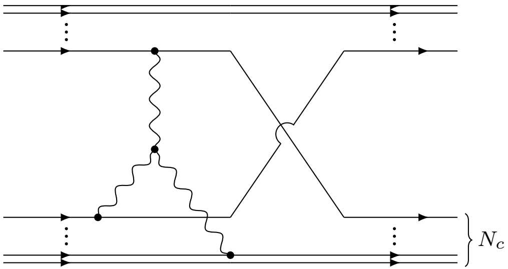
$i\frac{g_A}{2f_\pi}\frac{q^i}{\sqrt{2\omega_{\vec{q}}}}\tau^a\sigma^i$	$\frac{i}{f_\pi^2}\frac{\omega_{\vec{q}_1}-\omega_{\vec{q}_2}}{\sqrt{\omega_{\vec{q}_1}\omega_{\vec{q}_2}}}\ \epsilon^{abc}\tau^c$	$\frac{i}{f_\pi^2}\frac{\omega_{\vec{q}_1}+\omega_{\vec{q}_2}}{\sqrt{\omega_{\vec{q}_1}\omega_{\vec{q}_2}}}\ \epsilon^{abc}\tau^c$

Note that there is a relative sign for $\mathscr{H}_{22}$ depending on the direction of the pions

Large-N_c Baryon-Baryon Scattering

- → Baryons are defined as colorless N_c quark compound objects
- → Quantum numbers (spin, flavor) define "classical" equivalent baryon

Possible diagram contributing to baryon scattering



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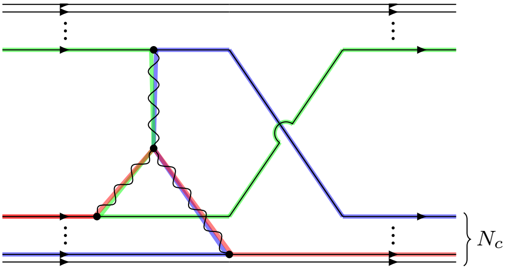
Possible diagram contributing to baryon scattering scaling as $N_c^{-2} \times N_c^3 = N_c$



⇒ Theory still without smooth large-N_c limit?

Nucleon Scattering on Quark Gluon Level

Nucleon-nucleon potential as *n*-quark operator matrix elements

$$V_{NN} = \sum_{n=1}^{\infty} \left\langle N_1' \, N_2' \,\middle|\, O^{(n)} \,\middle|\, N_2 \, N_1 \right\rangle$$

Scaling of operator expectation values

$$\left\langle N_1' \, N_2' \,\middle|\, O^{(n)} \,\middle|\, N_2 \, N_1 \right\rangle \sim f_{\text{Comb}}(N_c, n) f_{\text{Diag}}(N_c, n)$$

Combinatorical factor for large N_c are at most

$$f_{\text{Comb}}(N_c, n) = N_c(N_c - 1)(N_c - 2) \cdots (N_c - n) \sim N_c^n$$

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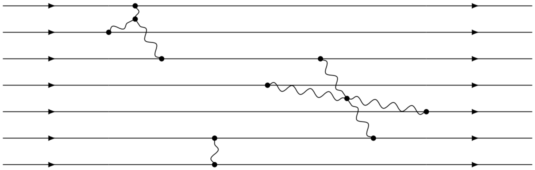
Example diagram for a 7-quark operator



$\Rightarrow$ Diagramtical factors are at most N_c^{1-n}

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Hartree Hamiltonian using $SU(4)$ one-quark operators

$$O^{(n)} = N_c \sum_{n_1, n_2 = 0}^{\infty} V^{(n)}_{n_1 n_2} \cdot \left(\frac{J}{N_c} \right)^{n_1} \cdot \left(\frac{I}{N_c} \right)^{n_2} \cdot \left(\frac{G}{N_c} \right)^{n - n_1 - n_2}$$

with matrix elements

$$\langle N' \, | \, I \, | \, N \rangle \sim N_c^0 \quad , \quad \langle N' \, | \, J \, | \, N \rangle \sim N_c^0 \quad , \quad \langle N' \, | \, G \, | \, N \rangle \sim N_c$$

Potential Using Fock-Space Transformations

- Start with Schrödinger equation $H\,|\Psi\rangle = E\,|\Psi\rangle$
- Introduce projection operators on Fock space

$$\eta\,|\Psi\rangle = |\Psi_\eta\rangle\,, \quad \lambda\,|\Psi\rangle = |\Psi_\lambda\rangle\,, \quad \eta^2 = \eta\,, \quad \lambda^2 = \lambda\,, \quad \eta\lambda = 0\,,$$

where $|\Psi_\eta\rangle$ is a purely nucleonic state

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  where $|\Psi_\eta\rangle$ is a purely nucleonic state
- Rewrite Schrödinger equation for subspaces

$$\begin{pmatrix} \eta H \eta & \eta H \lambda \\ \lambda H \eta & \lambda H \lambda \end{pmatrix} \begin{pmatrix} |\Psi_\eta\rangle \\ |\Psi_\lambda\rangle \end{pmatrix} = E \begin{pmatrix} |\Psi_\eta\rangle \\ |\Psi_\lambda\rangle \end{pmatrix}$$

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- Solve for effective nucleonic potential

$$\begin{aligned} V_{\text{eff}}(E)\,|\Psi_\eta\rangle &:= (E - H_0)\,|\Psi_\eta\rangle \\ &= \eta H_I \eta + \sum_{n=0} \eta H_I \lambda \frac{1}{E - H_0} \left(\lambda H_I \lambda \frac{1}{E - H_0} \right)^n \lambda H_I \eta \end{aligned}$$

Additional Unitary Transformations

Further transformation to make potential energy independent

$$\begin{pmatrix} |\eta\rangle \\ |\lambda\rangle \end{pmatrix} := U^\dagger \begin{pmatrix} |\Psi_\eta\rangle \\ |\Psi_\lambda\rangle \end{pmatrix} \Rightarrow U^\dagger H U \begin{pmatrix} |\eta\rangle \\ |\lambda\rangle \end{pmatrix} = \begin{pmatrix} H_\eta & 0 \\ 0 & H_\lambda \end{pmatrix} \begin{pmatrix} |\eta\rangle \\ |\lambda\rangle \end{pmatrix} = E \begin{pmatrix} |\eta\rangle \\ |\lambda\rangle \end{pmatrix}$$

Results in effective potential

$$V_{\text{eff}}^{\text{UT}} = \eta \left(\mathbb{1} + A^\dagger A \right)^{-1/2} \left(H + A^\dagger H + HA + A^\dagger HA \right) \left(\mathbb{1} + A^\dagger A \right)^{-1/2} \eta - H_0$$

Matrix U is depending on recursively defined operator $A = \lambda A \eta$

$$\lambda A \eta = \frac{\lambda}{E_\eta - E_\lambda} \left(H - [A, H] - AHA \right) \eta \, , \quad E_\eta := \eta H_0 \eta \, , \quad E_\lambda := \lambda H_0 \lambda$$

Additional Unitary Transformations

Further transformation to make potential energy independent

$$\begin{pmatrix} |\eta\rangle \\ |\lambda\rangle \end{pmatrix} := U^\dagger \begin{pmatrix} |\Psi_\eta\rangle \\ |\Psi_\lambda\rangle \end{pmatrix} \Rightarrow U^\dagger H U \begin{pmatrix} |\eta\rangle \\ |\lambda\rangle \end{pmatrix} = \begin{pmatrix} H_\eta & 0 \\ 0 & H_\lambda \end{pmatrix} \begin{pmatrix} |\eta\rangle \\ |\lambda\rangle \end{pmatrix} = E \begin{pmatrix} |\eta\rangle \\ |\lambda\rangle \end{pmatrix}$$

Results in effective potential

$$V^{\mathrm{UT}}_{\mathrm{eff}} = \eta \left(\mathbb{1} + A^\dagger A \right)^{-1/2} \left(H + A^\dagger H + HA + A^\dagger HA \right) \left(\mathbb{1} + A^\dagger A \right)^{-1/2} \eta - H_0$$

Matrix U is depending on recursively defined operator $A = \lambda A \eta$

$$\lambda A \eta = \frac{\lambda}{E_\eta - E_\lambda} \left(H - [A, H] - AHA \right) \eta \,, \quad E_\eta := \eta H_0 \eta \,, \quad E_\lambda := \lambda H_0 \lambda$$

Introduce perturbation scheme

$$H_I = \sum_{\kappa=1} H^{(\kappa)} \,, \qquad A = \sum_{\kappa=1} A^{(\kappa)} \,, \qquad O_1^{(\kappa_1)} O_2^{(\kappa_2)} = O_3^{(\kappa_1 + \kappa_2)}$$

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Results of $SU(4)_C$ Representation Theory

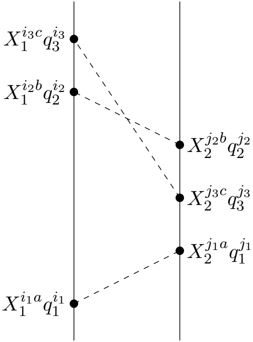
Dashen, Jenkins, and Manohar 1995

- Not possible to use basic techniques because subalgebra (X_0^{ia}) forms ideal
- Different irreducible representation according to strangeness
- Irreducible representations correspond to baryon towers $(I, J) = (n/2, n/2)$ for $n \in \mathbb{N}$
- It is possible for a pion to connect Δ-resonances and nucleons (and higher resonances with $|I' - I| \leq 1$ and $|J' - J| \leq 1$)
- For nucleonic matrix elements, $SU(4)_c$ symmetry is isomorphic to $SU(2)_J \otimes SU(2)_I \to SU(4)$
- For X_0^{ia} commutators to vanish, Δ-resonances must be included as internal states

Example diagram operator structure

$$V_{LCB}^{(4)} = V_{LCB}^{1112} + V_{LCB}^{1121} + V_{LCB}^{1211} + V_{LCB}^{2111}$$

$$V_{LCB}^{dcba} = \frac{1}{2} \eta \, H_{21}^{(1)} \frac{\lambda^1}{\omega_I^d} H_{21}^{(1)} \frac{\lambda^2}{(\omega_I + \omega_{II})^c} H_{21}^{(1)} \frac{\lambda^1}{\omega_I^b} H_{21}^{(1)} \, \eta \, H_{21}^{(1)} \frac{\lambda^1}{\omega_I^a} H_{21}^{(1)} \, \eta$$

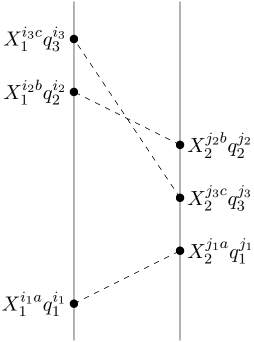


$$\propto X_1^{i_3 c} X_1^{i_2 b} X_1^{i_1 a} X_2^{j_2 b} X_2^{j_3 c} X_2^{j_1 a} q_1^{i_1} q_2^{i_2} q_3^{i_3} q_1^{j_1} q_2^{j_2} q_3^{j_3}$$

Example diagram operator structure

$$V^{(4)}_{LCB} = V^{1112}_{LCB} + V^{1121}_{LCB} + V^{1211}_{LCB} + V^{2111}_{LCB}$$

$$V^{dcba}_{LCB} = \frac{1}{2}\eta\, H^{(1)}_{21} \frac{\lambda^1}{\omega_I^d} H^{(1)}_{21} \frac{\lambda^2}{(\omega_I + \omega_{II})^c} H^{(1)}_{21} \frac{\lambda^1}{\omega_I^b} H^{(1)}_{21}\, \eta\, H^{(1)}_{21} \frac{\lambda^1}{\omega_I^a} H^{(1)}_{21}\, \eta$$



$$\begin{aligned} &\propto X_1^{i_3 c} X_1^{i_2 b} X_1^{i_1 a} X_2^{j_2 b} X_2^{j_3 c} X_2^{j_1 a} q_1^{i_1} q_2^{i_2} q_3^{i_3} q_1^{j_1} q_2^{j_2} q_3^{j_3} \\ &\times \frac{1}{2}\left(\frac{1}{\omega_1^2 \omega_2 \omega_3^3 (\omega_2 + \omega_3)^2} + \frac{1}{\omega_1^3 \omega_2 \omega_3^3 (\omega_2 + \omega_3)} \right. \\ &\qquad \left. + \frac{2}{\omega_1^2 \omega_2 \omega_3^4 (\omega_2 + \omega_3)} \right) \end{aligned}$$